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ENERGY CONSERVATION IN SPECIAL AND GENERALIZED SPECIAL RELATIVITY

Mubarak Ibrahim.S.Tagabo*, Mubarak Dirar A.Allah, Asim Ahmed Mohamed, Sawsan Ahmed Elhouri Ahmed

* Department of Physics- Faculty of Science- Sudan University of Science & Technology Khartoum-Sudan.
University of Africa College of Science-Department of Physics & Sudan University of Science & Technology-Departments of Physics- Khartoum - Sudan.
University of Barh -College of Applied & Industrial Sciences-Department of Physics - Khartoum -Sudan.

ABSTRACT
In this work the concept of energy and energy conservations in special relativity and generalized special relativity is discussed. The aim of the work is to incorporate potential energy in the energy relation. Some expressions conserves energy and other resembles Newtonian one, while other expressions combines Newtonian beside relativistic rest mass energy term.

KEYWORDS: Special relativity, energy, conservation special, relativity, generalized.

INTRODUCTION
Einstein special relativity (SR) is one of the biggest achievements that make radical modification to the concepts of space and time. It states that the space-time interval between two events is no longer constant for frames of referencies moving with constant velocity with respect to each other. As far as the conservation of energy and momentum results from the invariance of space and time, thus it is quite natural to expect SR new concepts to have a direct impact on energy momentum expression [1, 2]. The relativity of time, space, mass, and energy were verified and confirmed experimentally, these experiments comes as an ultimate reward confirming the key predictions of SR. Dispite these remarkable successes of SR, it suffers from noticeable Setbacks. For instance SR does not satisfy the correspondence principle in the sense that the expression of energy for SR does not reduced to the conventional Newtonian one. This is since the SR energy expression has no expression sensitive to the potential energy [3, 4]. Relation from a term taking care and feeling the existence of potential energy is in direct conflict with experiments and common sense. For instance, according to SR two particles moving with same velocity, one is in free space, and the other is in gravity field, have the same energy of course experiments and common sense shows that their energy are different [5]. This defect encourages some researchers to propose a generalized version of SR, called generalized SR (GSR). This new GSR has an energy term representing a potential energy and satisfies a Newtonian limit [6, 7]. This success of GSR encourages to use the conventional expression of kinetic and potential energy to see how energy conservation looks like in SR and GSR. This task is done in sections. Section 3 and 4 were devoted for discussion and conclusion

ENERGY CONSERVATION IN SR AND GSR
According to the very definition of potential energy V and Kinetic energy T, they can be defined in one dimension as:

$$F = -\frac{\partial V}{\partial x} = -\frac{dV}{dx} = \frac{dmv}{dt} \qquad (1)$$

$$T = \int \frac{dmv}{dt} dx = \int F.dx = -\int \frac{dV}{dx} dx$$

$$\int \frac{dmv}{dt} dx + c_3 = -\int \frac{dV}{dx} dx + c_4 = -V + c_4 \qquad (2)$$

$$T + c_3 = \int dmv \frac{dx}{dt} + c_3 = -V + c_4 \qquad (3)$$

$$T + V = c_4 - c_3 = c_5$$

$$\int v\, dmv = -V + c_5$$

$\int d(mv^2) - \int mvdv = -V + c_5$ (4)

Since: $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ (5)

And by defining

$$\cos\theta = {}^{v}/_{c} \qquad d\sin\theta = \cos\theta\, d\theta$$

Equation (4) reads

$$mv^2 + m_o c\int \frac{[\cos\theta][c\sin\theta\, d\theta]}{\sin\theta} = -V + c_5$$

$$mv^2 + m_o c^2 \int d\sin\theta = -V + c_5$$

$mv^2 + m_o c^2 \sin\theta = -V + c_5$ (6)

Utilizing equation (5) again

$\sin\theta = (1-\cos^2\theta)^{1/2} = (1 - v^2/c^2)^{1/2}$ (7)

Therefore

$$mv^2 + m_o c^2\,(1 - v^2/c^2)^{1/2} = -V + c_5$$

$$mv^2 + \frac{m_o c^2 (1 - v^2/c^2)}{\sqrt{(1 - v^2/c^2)}} = -V + c_5$$

$mv^2 + mc^2 - mv^2 = -V + c_5$

$mc^2 + V = c_5$ (8)

Thus in view of equation (3)

$T + V = c_5$ (9)

Thus the kinetic energy with in the frame work of SR is given by:

$$T = mc^2 \qquad (10)$$

Clearly the sum of kinetic energy and potential one is a constant of motion, i.e. it represents the total energy E which is conserved according to SR, thus

$$E = T + V = mc^2 + V \qquad (11)$$

$$E = m_0 c^2 \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} + \mathrm{V} \approx m_0 c^2 + \frac{1}{2}\, m_0 v^2 + V \qquad (12)$$

By redefining the energy to be:

$$E_N = E - m_0 c^2 = constant \qquad (13)$$

Then:

$$E_N = T + V \qquad (14)$$

Which is the ordinary Newton classical energy for generalized special relativistic (GSR)?

Then:

$$m = \frac{m_0}{\sqrt{1 - \frac{2\emptyset}{c^2} - \frac{v^2}{c^2}}} \qquad (15)$$

By defining:

$C_1 = 1 + \frac{2\emptyset}{c^2}$, ${C_2}^2 = C_1 c^2$ (16)

$$m = \frac{m_0}{\sqrt{C_1 - \frac{v^2}{c^2}}} = \frac{m_0}{\sqrt{C_1}\sqrt{1 - \frac{v^2}{C_1 c^2}}} = \frac{m_0}{\sqrt{C_1}\sqrt{1 - \frac{v^2}{{C_2}^2}}} \qquad (17)$$

In view of equation (4) and redefining (5) to be:

$$\csc\theta = \frac{v^2}{{C_2}^2} \qquad (18)$$

Equation (4) reads:

$$mv^2 + \frac{m_0{C_2}^2}{\sqrt{C_1}}\int d\sin\theta = -V + C_s$$

$$mv^2 + \frac{m_0{C_2}^2}{\sqrt{C_1}}\sin\theta = -V + C_s$$

$$mv^2 + \frac{m_0{C_2}^2}{\sqrt{C_1}}\sqrt{1-\cos^2\theta} = -V + C_s$$

$$mv^2 + \frac{m_0{C_2}^2}{\sqrt{C_1}}\sqrt{1-\frac{v^2}{C_1c^2}} = -V + C_s$$

$$mv^2 + \frac{m_0{C_2}^2}{\sqrt{C_1}}\sqrt{C_1-\frac{v^2}{c^2}} = -V + C_s$$

$$mv^2 + m_0{C_2}^2\sqrt{1+\frac{2\emptyset}{c^2}-\frac{v^2}{c^2}} = -V + C_s$$

$$mv^2 + mc^2\left(1+\frac{2\emptyset}{c^2}-\frac{v^2}{c^2}\right) = -V + C_s$$

$$mv^2 + mc^2 + 2m\emptyset - mv^2 = -V + c_5$$

$$mc^2 + 2m\emptyset + V = c_5 \qquad (19)$$

According to equation (3)

$$T + V = c_5 \qquad (20)$$

By defining V to be

$$V = -m\emptyset \qquad (21)$$

$$mc^2 - V = c_5 \qquad (22)$$

This equation is inconsistent with equation (20) even if one define the kinetic energy to be

$$T = mc^2 \qquad (23)$$

However one can redefine the relation between the force and kinetic energy to be

$$F = \frac{d\left({}^1/_2\,mv^2\right)}{dx} = \frac{dT}{dx} \qquad (24)$$

For particles having constant mass:

$$F = \frac{1}{2}m\frac{dv^2}{dx} = mv\frac{dv}{dx} = mv\frac{dv}{dt}\frac{dt}{dx} = m\frac{v}{v}\frac{dv}{dt} = m\frac{dv}{dt} \quad (25)$$

Hence the definition of kinetic energy in terms of the force is consistent with the formal definition of force for particles having constant mass.
Thus according to this definition (24) together with definition (1):

$$F = \frac{d(T)}{dx} = -\frac{dV}{dx} \quad (26)$$

There force:

$$\frac{d(T+V)}{dx} = 0$$

Let:

$$E = T + V = \frac{1}{2}mv^2 + V \qquad (27)$$

$$\frac{dE}{dx} = 0 \qquad (28)$$
$$E = T + V = \ constant \qquad (29)$$

However for T based on relation (10) and (24) requires:

$$F = -\frac{d\left(\frac{1}{2}mv^2 - \frac{1}{2}mc^2\right)}{dx} = \frac{dT}{dx} = -\frac{dV}{dx} = -\frac{\mathrm{d}V}{\mathrm{d}x} = -\frac{\mathrm{d}m\emptyset}{\mathrm{d}x} \quad (30)$$

Since: $c > v$
Thus for positive T
One can define:

$$T = \frac{1}{2}mc^2 - \frac{1}{2}mv^2 \qquad (31)$$

Where $\quad T > 0 \quad as\ far\ as \quad c > v$
Thus:

$$\frac{d\left(m\emptyset + \frac{1}{2}mc^2 - \frac{1}{2}mv^2\right)}{dx} = 0$$

This requires:

$$T + V = m\emptyset + \frac{1}{2}mc^2 - \frac{1}{2}mv^2 = C_0 = constant\ (32)$$

$$\frac{\frac{c^2}{2}m_0\left(\frac{2\emptyset}{c^2} + 1 - \frac{v^2}{c^2}\right)}{\sqrt{1 + \frac{2\emptyset}{c^2} - \frac{v^2}{c^2}}} = C_0$$

$$\frac{c^2}{2}m_0\left(1 + \frac{2\emptyset}{c^2} - \frac{v^2}{c^2}\right)^{\frac{1}{2}} = C_0$$

$$\frac{c^2}{2}m_0\left(1 + \frac{2\emptyset}{c^2} - \frac{v^2}{c^2}\right)^{\frac{1}{2}} = C_0$$

$$\frac{m_0^{\frac{1}{2}}}{\sqrt{2}}\left(\frac{1}{2}m_0c^2 + m_0\emptyset - \frac{1}{2}m_0v^2\right) \qquad (33)$$

Squiring both sides:

$$\frac{1}{2}m_0c^2 + m_0\emptyset - \frac{1}{2}m_0v^2 = \frac{2{C_0}^2}{m_0c^2}$$

$$\frac{1}{2}m_0c^2 - \frac{1}{2}m_0v^2 + m_0\emptyset = \frac{2{C_0}^2}{m_0c^2} = constant \quad (34)$$

To make this consistent with the fact that rest mass energy should exit in any relativistic expression, one can suppose that:

$$\frac{2{C_0}^2}{m_0c^2} = C_1 \qquad (35)$$

To get:

$$\frac{1}{2}m_0c^2 - \frac{1}{2}m_0v^2 + m_0\emptyset = C_1 \qquad (36)$$

Thus according to relation (30):

$$T_0 + V_0 = C_1 \qquad (37)$$

With the aid of equations (31), (34) and (36) requires:

$$T + V = C_0 = C_1 = \frac{2{C_0}^2}{m_0c^2} = T_0 + V_0 \quad (38)$$

Thus the energy conservation requires:

$$2C_0 = m_0c^2 \qquad C_0 = \frac{1}{2}\,m_0c^2\ (39)$$

Another approach can be tackled by assuming the kinetic energy T and the potential energy to be defined by:

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$$T = \int \frac{d(mv)}{dt} dx = \int dmv \frac{dx}{dt} = \int vdmv = \int Fdx$$
$$= \int d(mv^2) - \int mvdv = mv^2 - \int \frac{1}{2} mdv^2 \quad (40)$$
$$- \int Fdx = V = \int dV = \int md\emptyset \quad (41)$$
$$\int Fdx = - \int md\emptyset = mv^2 - \frac{1}{2} \int mdv^2 = C_0$$
$$T + V = C_0 = \text{constant} \quad (42)$$
$$mv^2 + \int md \left[\emptyset - \frac{v^2}{2} \right] = C_0$$
$$mv^2 - \frac{c^2}{2} \int md \left[\frac{v^2}{c^2} - \frac{2\emptyset}{c^2} \right] = C_0$$

Defining:

$$\cos^2 \theta = z^2 = \frac{v^2}{c^2} - \frac{2\emptyset}{c^2} \quad (43)$$

One gets:

$$m = \frac{m_0}{\sqrt{1 - \cos^2\theta}} = \frac{m_0}{\sin \theta} \quad (44)$$
$$mv^2 - \frac{c^2}{2} \int \frac{m_0 dz^2}{\sin \theta} = mv^2 - \frac{c^2 m_0}{2} \int \frac{2zdz}{\sin \theta} = C_0$$
$$mv^2 + m_0 c^2 \int \frac{\cos \theta \sin \theta \, d\theta}{\sin \theta} = C_0$$
$$mv^2 + m_0 c^2 \sin \theta = C_0$$
$$mv^2 + m_0 c^2 \sqrt{1 - \cos^2\theta} = C_0$$
$$mv^2 + m_0 c^2 \sqrt{1 - z^2} = C_0$$
$$mv^2 + \frac{m_0 c^2 (1 - z^2)}{\sqrt{1 - z^2}} = C_0$$
$$mv^2 + mc^2 \left(1 + \frac{2\emptyset}{c^2} - \frac{v^2}{c^2} \right) = C_0$$
$$mv^2 + mc^2 + 2\mathrm{m}\emptyset - \mathrm{m}v^2 = C_0$$
$$mc^2 + 2\mathrm{m}\emptyset = C_0 \quad (45)$$

Thus the energy conservation requires:

$$E = T + V = mc^2 + 2\mathrm{m}\emptyset \quad (46)$$

In the classical limit, when:

$$v \ll V$$
$$E = m_0 \left(1 + \frac{2\emptyset}{c^2} - \frac{v^2}{c^2} \right)^{-\frac{1}{2}} c^2 + 2m_0\emptyset \left(1 - \frac{2\emptyset}{c^2} - \frac{v^2}{c^2} \right)^{-\frac{1}{2}}$$
$$= m_0 \left(1 - \frac{\emptyset}{c^2} + \frac{1}{2}\frac{v^2}{c^2} \right) c^2 + 2m_0\emptyset \left(1 - \frac{\emptyset}{c^2} + \frac{1}{2}\frac{v^2}{c^2} \right)$$
$$E = m_0 c^2 - m_0\emptyset + \frac{1}{2} m_0 v^2 + 2m_0\emptyset$$
$$= m_0 c^2 + \frac{1}{2} m_0 v^2 + m_0\emptyset$$

$E = m_0 c^2 + T + V$ (47)

Which is the conventional ordinary Newton energy relation with additional term standing for rest mass energy.

DISCUSSION

Many relativistic expressions for energy that satisfies energy conservation were discussed. In the first one the ordinary SR expression for mass in equation (4), beside the ordinary definition of force in equation (1) were used to find expression E- the kinetic energy T be equal to mc^2. T is not like Newtonian one, but for small v

$$T = m_0c^2 + \frac{1}{2}\,m_0v^2$$

Thus it resembles Newtonian one with additional rest mass energy term. The SR energy reduces to Newtonian one in eq. (12) and is conserved. It is more is more advance than SR one since it is conserved and consists of potential term. Another E is based on GSR beside expression for (m) in equation (15). The energy is conserved according to equation (22) but V becomes with a minus sign.
Anew definition of force in equation (24) with ordinary expression for T is also used to define conservative E. the potential is related to F in a conventional way. Conservation was satisfied in equation (32) but T is defined in different way, i.e.

$$T = \frac{1}{2}mc^2 - \frac{1}{2}\,mv^2$$
$$T > 0 \qquad since \qquad c > v$$

CONCLUSIONS

Special relativity (SR) and GSR energy having kinetic and potential term can be conserved if one utilizes the formal definition of force. All expressions for energy reduces to New tonion one.

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